(50)

$tg\,\alpha = 3$

a) si $\alpha = 1^{er}$ cuad. / $2\alpha \to 2^{o}$ cuad

$\cos\alpha = \sqrt{\dfrac{1}{1+tg^2\alpha}} \leftarrow \boxed{tg^2\alpha + 1 = \dfrac{1}{\cos^2\alpha}}$

$\cos\alpha = \dfrac{1}{\sqrt{10}} = \dfrac{\sqrt{10}}{10}$

$sen\,\alpha = tg\alpha . \cos\alpha = 3\,\dfrac{\sqrt{10}}{10} = \dfrac{3\sqrt{10}}{10}$

$sen\,2\alpha = 2 . sen\alpha . \cos\alpha = \dfrac{6 . 10}{100} = 0{,}6$

$\cos 2\alpha = \cos^2\alpha - sen^2\alpha = -0{,}8$

$tg\,2\alpha = -0{,}75$

b) $\alpha = 3^{er}$ cuad.

$tg\,\alpha = 3 > 1$

$225° < \alpha < 270°$

$\downarrow$

$2\alpha \to 2^{o}$ cuad.

= resultados que a)

(59)

(i) $sen^2\alpha - sen^2\beta = sen(\alpha+\beta) . sen(\alpha-\beta)$

$sen(\alpha+\beta) . sen(\alpha-\beta) = (sen\alpha . \cos\beta + \cos\alpha . sen\beta) . (sen\alpha \cos\beta - sen\beta\cos\alpha)$

$= sen^2\alpha . \cos^2\beta - \cos^2\alpha . sen^2\beta = sen^2\alpha . \cos^2\beta - (1 - sen^2\alpha)\,sen^2\beta =$

$= sen^2\alpha . \cos^2\beta - sen^2\beta + sen^2\beta . sen^2\alpha \underset{f. común}{=} sen^2\alpha\,\underbrace{(\cos^2\beta + sen^2\beta)}_{1} - sen^2\beta$

$= \underline{sen^2\alpha - sen^2\beta}$

f) $\dfrac{1-\cos 2\alpha}{2\,sen\,\alpha} - \dfrac{sen\,2\alpha}{1+\cos 2\alpha} = sen\alpha - tg\,\alpha$

$\dfrac{1 - \cos^2\alpha + sen^2\alpha}{2\,sen\alpha} - \dfrac{2\,sen\alpha . \cos\alpha}{1 + \cos^2\alpha - sen^2\alpha} = \dfrac{2\,sen^2\alpha}{2\,sen\alpha} - \dfrac{2\,sen\alpha . \cos\alpha}{2\cos^2\alpha}$

$= sen\alpha - tg\,\alpha.$

g) $\dfrac{sen\,2\alpha}{1+\cos 2\alpha} = tg\,\alpha$

$\dfrac{2\,sen\alpha . \cos\alpha}{1 + \cos^2\alpha - sen^2\alpha} = \dfrac{\cancel{2}\,sen\alpha . \cos\alpha}{2\cos^2\alpha} = tg\,\alpha.$

ii) $tg\left(\frac{\pi}{4}+\alpha\right) - tg\left(\frac{\pi}{4}-\alpha\right) = 2\,tg\,2\alpha.$

$$= \frac{tg\frac{\pi}{4} + tg\,\alpha}{1 - tg\frac{\pi}{4}\cdot tg\,\alpha} - \frac{tg\frac{\pi}{4} - tg\,\alpha}{1 + tg\frac{\pi}{4}\cdot tg\,\alpha} = \frac{1+tg\,\alpha}{1-tg\,\alpha} - \frac{1-tg\,\alpha}{1+tg\,\alpha}$$

$$\frac{(1+tg\,\alpha)^2 - (1-tg\,\alpha)^2}{1-tg^2\alpha} = \frac{\cancel{1} + \cancel{tg^2\alpha} + 2tg\,\alpha - \cancel{1} - \cancel{tg^2\alpha} + 2tg\,\alpha}{1-tg^2\alpha}$$

$$= \frac{4\,tg\,\alpha}{1-tg^2\alpha} = 2\cdot\left(\frac{tg\,\alpha}{1-tg^2\alpha}\right) = 2\cdot tg\,2\alpha.$$

65

a) $tg\,x + 4\,cotg\,x = 5$

$tg\,x + 4\cdot\frac{1}{tg\,x} = 5$

$tg^2x + 4 - 5\,tg\,x = 0$

$tg^2x - 5\,tg\,x + 4 = 0$

$tg\,x = \frac{5 \pm \sqrt{25-16}}{2}$

$tg\,x = 4$ → $x = 75°58'$, $x = 255°58'$

$tg\,x = 1$ → $x = 45°$, $x = 225°$

b) $8\cos 2x = 8\cos x - 9$

$8(\cos^2x - sen^2x) - 8\cos x + 9 = 0$

$8\cos^2x - 8sen^2x - 8\cos x + 9 = 0$

$8\cos^2x - 8(1-\cos^2x) - 8\cos x + 9 = 0$

$8\cos^2x - 8 + 8\cos^2x - 8\cos x + 9 = 0$

$16\cos^2x - 8\cos x + 1 = 0$

$\cos x = \frac{8 \pm \sqrt{64-24}}{32} = 1/4$

~~x = arc tg 1~~ $x = 75°31'$, $x = 284°29'$

c) $tg\,2x = cotg\,x$

$\frac{2\,tg\,x}{1-tg^2x} = \frac{1}{tg\,x}$

$2\,tg^2x = 1 - tg^2x$

$3\,tg^2x = 1$

$tg^2x = \frac{1}{3}$ $\quad tg\,x = \pm\frac{\sqrt{3}}{3}$

$x = 30°$, $x = 210°$

$x = 150°$ $\quad x = 330°$

d) $2\,sen^2x + \cos 2x = 4\cos^2x$

$2\,sen^2x + \cos^2x - sen^2x = 4\cos^2x$

$\underbrace{sen^2x + \cos^2x}_{1} - 4\cos^2x = 0$

$4\cos^2x = 1$

$\cos^2x = \frac{1}{4}$ $\quad \cos x = \pm 1/2$

$x = 60°$ $\quad x = 300°$

$x = 120°$ $\quad x = 240°$

3.60

a) $(\operatorname{sen}\alpha+\cos\alpha)^2+(\operatorname{sen}\alpha-\cos\alpha)^2=\operatorname{sen}^2\alpha+\cos^2\alpha+\cancel{2\operatorname{sen}\alpha\cdot\cos\alpha}+\operatorname{sen}^2\alpha+\cos^2\alpha-\cancel{2\operatorname{sen}\alpha\cdot\cos\alpha}=$

$1+1=2.$

c) $\dfrac{1-\operatorname{tg}^2\alpha}{1+\operatorname{tg}^2\alpha}=\dfrac{1-\dfrac{\operatorname{sen}^2\alpha}{\cos^2\alpha}}{1+\dfrac{\operatorname{sen}^2\alpha}{\cos^2\alpha}}=\dfrac{\dfrac{\cos^2\alpha-\operatorname{sen}^2\alpha}{\cos^2\alpha}}{\dfrac{\cos^2\alpha+\operatorname{sen}^2\alpha}{\cos^2 x}}=\cos^2\alpha-\operatorname{sen}^2\alpha=\underline{\underline{\cos 2\alpha}}.$

b) $\operatorname{tg}\alpha\cdot\operatorname{tg}\beta\,(\operatorname{cotg}\alpha+\operatorname{cotg}\beta)=\operatorname{tg}\alpha\cdot\operatorname{tg}\beta\cdot\left(\dfrac{1}{\operatorname{tg}\alpha}+\dfrac{1}{\operatorname{tg}\beta}\right)=\cancel{\operatorname{tg}\alpha\cdot\operatorname{tg}\beta}\left(\dfrac{\operatorname{tg}\beta+\operatorname{tg}\alpha}{\cancel{\operatorname{tg}\alpha\cdot\operatorname{tg}\beta}}\right)=$

$=\operatorname{tg}\alpha+\operatorname{tg}\beta$

d) $\dfrac{\cos^2\alpha}{1-\operatorname{sen}\alpha}=\dfrac{1-\operatorname{sen}^2\alpha}{1-\operatorname{sen}\alpha}=\dfrac{(\cancel{1-\operatorname{sen}\alpha})(1+\operatorname{sen}\alpha)}{\cancel{1-\operatorname{sen}\alpha}}=1+\operatorname{sen}\alpha.$

e) $\operatorname{sen}2\alpha\,(\operatorname{tg}\alpha+\operatorname{cotg}\alpha)=2\operatorname{sen}\alpha\cdot\cos\alpha\left(\dfrac{\operatorname{sen}\alpha}{\cos\alpha}+\dfrac{\cos\alpha}{\operatorname{sen}\alpha}\right)=2\cancel{\operatorname{sen}\alpha\cdot\cos\alpha}\,\dfrac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\cancel{\operatorname{sen}\alpha\cdot\cos\alpha}}=\underline{\underline{2}}$

f) $\dfrac{\cos^2\alpha}{1-\cos\alpha}\cdot\dfrac{\operatorname{sen}^2\alpha}{1-\operatorname{sen}\alpha}=\dfrac{(1-\operatorname{sen}^2\alpha)(1-\cos^2\alpha)}{(1-\cos\alpha)(1-\operatorname{sen}\alpha)}=\dfrac{(\cancel{1-\operatorname{sen}\alpha})(1+\operatorname{sen}\alpha)(\cancel{1-\cos\alpha})(1+\cos\alpha)}{(\cancel{1-\cos\alpha})(\cancel{1-\operatorname{sen}\alpha})}=$

$=(1+\operatorname{sen}\alpha)(1+\cos\alpha)$

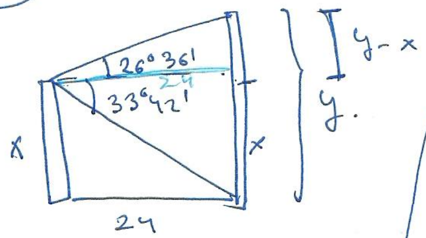
g) $\dfrac{\operatorname{sen}\alpha\cdot\cos\alpha}{\cos^2\alpha-\operatorname{sen}^2\alpha}\cdot\dfrac{1-\operatorname{tg}^2\alpha}{\operatorname{tg}\alpha}\overset{⊛}{=}\dfrac{\overset{\rightarrow\ \frac{\operatorname{sen}2\alpha}{2}}{\operatorname{sen}\alpha\cdot\cos\alpha}}{\cos 2\alpha}\cdot\dfrac{1}{\dfrac{\operatorname{tg}2\alpha}{2}}=$

$\dfrac{\operatorname{sen}2\alpha}{2\cdot\cos 2\alpha}\cdot\dfrac{2}{\operatorname{tg}2\alpha}=\dfrac{2\operatorname{tg}2\alpha}{2\operatorname{tg}2\alpha}=\underline{\underline{1}}$

⊛ $\operatorname{tg}2\alpha=\dfrac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$

$\dfrac{\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}=\dfrac{\operatorname{tg}2\alpha}{2}$

3.77



$$\operatorname{tg} 26^\circ 36' = \frac{y-x}{24}$$

$$\operatorname{tg} 33^\circ 42' = \frac{x}{24.}$$

$$x = 24 \cdot \operatorname{tg} 33^\circ 42' = 16\,m.$$

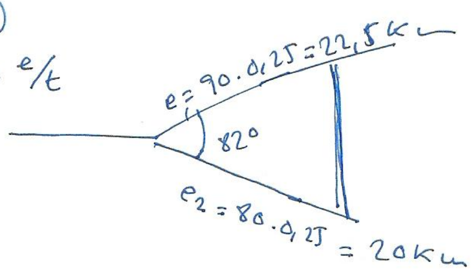
$$y - x = \operatorname{tg} 26^\circ 36'$$

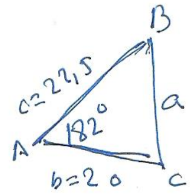
$$y - x = 12$$

$$y = 12 + 16 = \underline{28\,m}$$

3.78

$v = e/t$

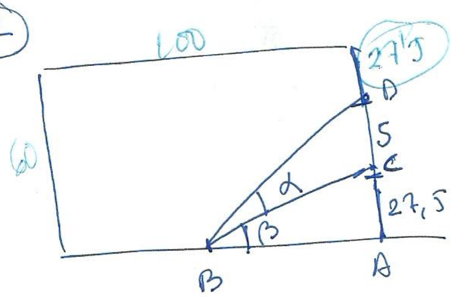




$$a^2 = 22{,}5^2 + 20^2 - 2 \cdot 22{,}5 \cdot 20 \cdot \cos 82^\circ$$

$$a = \underline{27{,}9\ km}$$

3.82



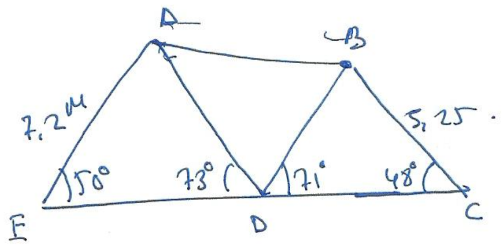
$$\operatorname{tg} \beta = \frac{27{,}5}{50} = 0{,}55 \rightarrow \beta = 28{,}81^\circ$$

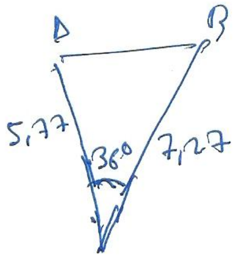
B.

$$\operatorname{tg}(\alpha + \beta) = \frac{32{,}5}{50} \longrightarrow \alpha + \beta = 33{,}02^\circ$$

$$\alpha = 33{,}02^\circ - 28{,}81^\circ = 4{,}21^\circ$$

3.83.





$\frac{AD}{\operatorname{sen} 50°} = \frac{7,2}{\operatorname{sen} 73°} \Rightarrow \overline{AD} = 5,77 \text{ m}.$

$\frac{BD}{\operatorname{sen} 48°} = \frac{5,25}{\operatorname{sen} 71°} \Rightarrow \overline{BD} = 7,27 \text{ m}.$

$\overline{AB}^2 = 5,77^2 + 7,27^2 - 2 \cdot 5,77 \cdot 7,27 \cdot \cos 36°$

$\overline{AB} = 4,27 \text{ m}$

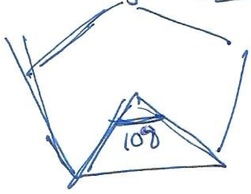
3.84

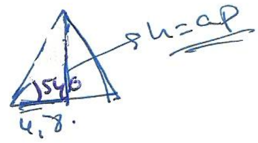
$\boxed{144 \text{ cm}^2}$ $\sqrt{144} = 12$ cm lado

$P = 12 \cdot 4 = 48$ cm

$P = 48$ cm.

$l = \frac{48}{5} = 9,6$ cm.





$S_{Ang} = 180°(n-2) = 180 \cdot 3 = 540°$

$\operatorname{tg} 54° = \frac{h}{4,8} \Rightarrow h = 6,58$ cm

$\frac{48 \cdot 6,58}{2}$

$A = \frac{P \cdot ap}{2} = 157,92 \text{ cm}^2$