EJERCICIOS PARA PRACTICAR DERIVADAS HOJA 1

FUNCIONES	DERIVADAS
a) $f(x)=\dfrac{1-x}{1+x}$	$f'(x)=\dfrac{(-1).(1+x)-(1-x).1}{(1+x)^2}=\dfrac{-1-x-1+x}{(1+x)^2}=\dfrac{-2}{(1+x)^2}$
b) $f(x)=\sqrt{\dfrac{1-x}{1+x}}$	$f'(x)=\dfrac{1}{2\sqrt{\dfrac{1-x}{1+x}}}.D\left(\dfrac{1-x}{1+x}\right)=\dfrac{1}{2}\sqrt{\dfrac{1+x}{1-x}}.\dfrac{-2}{(1+x)^2}=$ $=-\sqrt{\dfrac{1+x}{1-x}}.\dfrac{1}{(1+x)^2}$
c) $f(x)=\ln\dfrac{1-x}{1+x}$	$f'(x)=\dfrac{1+x}{1-x}.D\left(\dfrac{1-x}{1+x}\right)=\dfrac{1+x}{1-x}.\dfrac{-2}{(1+x)^2}=$ $=-\dfrac{2}{1-x^2}$
d) $f(x)=\dfrac{1-\tan x}{1+\tan x}$	$f'(x)=\dfrac{-(1+\tan^2 x)(1+\tan x)-(1+\tan^2 x)(1-\tan x)}{(1+\tan x)^2}=$ $=\dfrac{-1-\tan x-\tan^2 x-\tan^3 x-1+\tan x-\tan^2 x+\tan^3 x}{(1+\tan x)^2}=$ $=\dfrac{-2-2\tan^2 x}{(1+\tan x)^2}=\dfrac{-2(1+\tan x)^2}{(1+\tan x)^2}=-2$
e) $f(x)=\sqrt{\dfrac{1-\tan x}{1+\tan x}}$	$f'(x)=\dfrac{1}{2\sqrt{\dfrac{1-\tan x}{1+\tan x}}}.D\left(\dfrac{1-\tan x}{1+\tan x}\right)=\dfrac{1}{2}\sqrt{\dfrac{1+\tan x}{1-\tan x}}.(-2)=$ $=-\sqrt{\dfrac{1+\tan x}{1-\tan x}}$
f) $f(x)=\ln\sqrt{e^{\tan x}}$	$f'(x)=\dfrac{1}{\sqrt{e^{\tan x}}}.\dfrac{1}{2\sqrt{e^{\tan x}}}.e^{\tan x}.(1+\tan^2 x)=\dfrac{1+\tan^2 x}{2}$
g) $f(x)=\sqrt{3^{x+1}}$	$f'(x)=\dfrac{1}{2\sqrt{3^{x+1}}}.3^{x+1}.\ln 3.1=\dfrac{3^{x+1}.\ln 3}{2\sqrt{3^{x+1}}}=\dfrac{\ln 3.\sqrt{3^{x+1}}}{2}$
h) $f(x)=\log(\sin x.\cos x)^2$	$f'(x)=\dfrac{1}{(\sin x.\cos x)^2}.\dfrac{1}{\ln 10}.2(\sin x.\cos x).$ $.(\cos^2 x-\sin^2 x)=\dfrac{1}{\ln 10}.\dfrac{4.(2.\sin x.\cos x).\cos 2x}{(2.\sin x.\cos x)^2}=$ $=\dfrac{4\cos 2x}{\ln 10.\sin 2x}=\dfrac{4}{\ln 10.\tan 2x}$

i)	$f(x)=\sin^2 x+\cos^2 x+x$	$f(x)=1+x$ à $f'(x)=1$
j)	$f(x)=\sin\sqrt{x+1}.$ $.\cos\sqrt{x-1}$	$f'(x)=\left(\cos\sqrt{x+1}\right).\dfrac{1}{2\sqrt{x+1}}.\cos\sqrt{x-1}+$ $+\sin\sqrt{x+1}.\left(-\sin\sqrt{x-1}\right).\dfrac{1}{2\sqrt{x-1}}=$ $=\dfrac{\cos\sqrt{x+1}.\cos\sqrt{x-1}}{2\sqrt{x+1}}-\dfrac{\sin\sqrt{x+1}.\sin\sqrt{x-1}}{2\sqrt{x-1}}$
k)	$f(x)=\arcsin\sqrt{x}$	$f'(x)=\dfrac{1}{\sqrt{1-x}}.\dfrac{1}{2\sqrt{x}}$

l)	$f(x)=$ $=\sin\left(3x^5-2\sqrt{x}+\sqrt[3]{2x}\right)$	$f'(x)=\cos\left(3x^5-2\sqrt{x}+\sqrt[3]{2x}\right).$ $.\left(15x^4-\dfrac{1}{\sqrt{x}}+\dfrac{2}{3\sqrt[3]{4x^2}}\right)$
m)	$f(x)=\sqrt{\sin x+x^2+1}$	$f'(x)=\dfrac{\cos x+2x}{2\sqrt{\sin x+x^2+1}}$
n)	$f(x)=\cos^2\sqrt[3]{x+(3-x)^2}$	$f'(x)=2\cos\sqrt[3]{x+(3-x)^2}.$ $.\left[-\sin\sqrt[3]{x+(3-x)^2}\right]\dfrac{1+2(3-x)(-1)}{3\sqrt[3]{\left[x+(3-x)^2\right]^2}}=$ $=\dfrac{\sin\left[2\sqrt[3]{x+(3-x)^2}\right].(-5+2x)}{3\sqrt[3]{\left[x+(3-x)^2\right]^2}}$

FUNCIONES	DERIVADAS
1 a) $y=\dfrac{x^2+3}{x^2-2}$	$y'=\dfrac{2x\left(x^2+3\right)-2x\left(x^2-3\right)}{\left(x^2+3\right)^2}=\dfrac{12x}{\left(x^2+3\right)^2}$
1 b) $y=\sqrt[3]{3x^2}$	$y'=\dfrac{6x}{3\sqrt[3]{9x^4}}=\dfrac{2}{\sqrt[3]{9x}}$
2 a) $y=\left(\dfrac{1-x}{1+x}\right)^{2/3}$	$y'=\dfrac{2}{3}\left(\dfrac{1-x}{1+x}\right)^{-1/3}.\dfrac{-1.(1+x)-(1-x).1}{(1+x)^2}=$ $=\dfrac{2}{3}.\sqrt[3]{\dfrac{1+x}{1-x}}.\dfrac{-2}{(1+x)^2}$
2 b) $y=\dfrac{2}{x}+\dfrac{x^2}{2}$	$y=2x^{-1}+\dfrac{1}{2}x^2$ à $y'=-2x^{-2}+x=-\dfrac{2}{x^2}+x$
3 a) $y=\dfrac{\ln x}{x}$	$y'=\dfrac{\dfrac{1}{x}.x-\ln x}{x^2}=\dfrac{1-\ln x}{x^2}$
3 b) $y=7e^{-x}$	$y'=7e^{-x}(-1)=-7e^{-x}$
4 a) $y=\dfrac{e^x+e^{-x}}{e^x-e^{-x}}$	$y'=\dfrac{\left(e^x-e^{-x}\right)\left(e^x-e^{-x}\right)-\left(e^x+e^{-x}\right)\left(e^x+e^{-x}\right)}{\left(e^x-e^{-x}\right)^2}=$ $=\dfrac{\left(e^x-e^{-x}\right)^2-\left(e^x+e^{-x}\right)^2}{\left(e^x-e^{-x}\right)^2}=$ $=\dfrac{\left(e^x-e^{-x}+e^x+e^{-x}\right)\left(e^x-e^{-x}-e^x-e^{-x}\right)}{\left(e^x-e^{-x}\right)^2}=$
4 b) $y=\sin x.\cos x$	$y'=\cos^2 x-\sin^2 x=\cos 2x$
5 a) $y=\dfrac{1}{\sin x}$	$y=(\sin x)^{-1}$ à $y'=-(\sin x)^{-2}.\cos x=-\dfrac{\cos x}{\sin^2 x}$
5 b) $y=\ln\left(x^2+1\right)$	$y'=\dfrac{2x}{x^2+1}$
6 a) $y=\arctan\dfrac{x}{3}$	$y'=\dfrac{1/3}{1+(x/3)^2}=\dfrac{1}{3\left(1+\dfrac{x^2}{9}\right)}=\dfrac{3}{9+x^2}$
6 b) $y=\cos^2(2x-\pi)$	$y'=2\cos(2x-\pi).\left[-\sin(2x-\pi)\right].2=$ $=-2\sin(4x-2\pi)$
7 a) $y=\sin^2 x$	$y'=2\sin x.\cos x=\sin 2x$
7 b) $y=\sqrt{\tan x}$	$y'=\dfrac{1+\tan^2 x}{2\sqrt{\tan x}}$
8 a) $y=\sin x^2$	$y'=\left(\cos x^2\right).2x$
8 b) $y=\arctan\left(x^2+1\right)$	$y'=\dfrac{2x}{1+\left(x^2+1\right)^2}$

9 a) $y=\left(2\sqrt{x}-3\right)^7$	$y'=7\left(2\sqrt{x}-3\right)^6.\frac{2}{2\sqrt{x}}=\frac{7\left(2\sqrt{x}-3\right)^6}{\sqrt{x}}$
9 b) $y=\log_2\sqrt{x}$	$y'=\frac{1}{\log_2\sqrt{x}}.\frac{1}{\ln 2}.\frac{1}{2\sqrt{x}}=\frac{1}{2\sqrt{x}.\ln 2.\log_2\sqrt{x}}$
10 a) $y=\sin^2 x^2$	$y'=2\sin x^2.\cos x^2.2x=2x.\sin 2x^2$
10 b) $y=\arctan\frac{1}{x}$	$y'=\frac{-\frac{1}{x^2}}{1+\frac{1}{x^2}}=-\frac{1}{x^2+1}$
11 a) $y=\cos^5\left(7x^2\right)$	$y'=5\cos^4\left(7x^2\right).\left[-\sin\left(7x^2\right)\right].14x$
11 b) $y=3^x+1$	$y'=3^x.\ln 3$
12 a) $y=\sqrt[3]{(5x-3)^2}$	$y=(5x-3)^{2/3}$ à $y'=\frac{2}{3}(5x-3)^{-1/3}.5=\frac{10}{3\sqrt[3]{5x-3}}$
12 b) $y=\arcsin\frac{x^2}{3}$	$y'=\frac{1}{\sqrt{1-\frac{x^4}{9}}}.\frac{2x}{3}=\frac{2x}{\sqrt{9-x^4}}$
13 a) $y=\ln(2x-1)$	$y'=\frac{2}{2x-1}$
13 b) $y=\tan\frac{x^2}{2}$	$y'=\left(1+\tan^2\frac{x^2}{2}\right).x$
14 a) $y=\ln\left(x^2-1\right)$	$y'=\frac{2x}{x^2-1}$
14 b) $y=\arccos\sqrt{2x}$	$y'=\frac{1}{\sqrt{1-4x^2}}.\frac{2}{2\sqrt{2x}}=\frac{1}{\sqrt{1-4x^2}\sqrt{2x}}$
15 a) $y=\ln\sqrt{1-x}$	$y'=\frac{1}{\sqrt{1-x}}.\frac{-1}{2\sqrt{1-x}}=-\frac{1}{2(1-x)}$
15 b) $y=(\arctan x)^2$	$y'=2(\arctan x)\frac{1}{1+x^2}=\frac{2(\arctan x)}{1+x^2}$
16 a) $y=\log_3(7x+2)$	$y'=\frac{1}{7x+2}.\frac{1}{\ln 3}.7=\frac{7}{(7x+2)\ln 3}$
16 b) $y=\ln\tan\frac{3}{x}$	$y'=\frac{1}{\tan\frac{3}{x}}.\left(1+\tan^2\frac{3}{x}\right)\left(-\frac{3}{x^2}\right)$
17 a) $y=e^{4x}$	$y'=e^{4x}.4$
17 b) $y=\ln\left(\ln\frac{1}{x}\right)$	$y'=\frac{1}{\ln\frac{1}{x}}.x.\left(-\frac{1}{x^2}\right)=-\frac{1}{x\ln x}$

18 a)	$y = 2^x$	$y' = 2^x.\ln 2$
18 b)	$y = \arcsin\dfrac{x+1}{x-1}$	$y' = \dfrac{1}{\sqrt{1-\left(\dfrac{x+1}{x-1}\right)^2}}.\dfrac{(x-1)-(x+1)}{(x-1)^2} =$ $= \dfrac{x-1}{\sqrt{(x-1)^2-(x+1)^2}}.\dfrac{-2}{(x-1)^2} =$ $= \dfrac{-2}{(x-1)^2\sqrt{2x.(-2)}} = \dfrac{-1}{(x-1)^2\sqrt{-x}}$
19 a)	$y = 5\tan^3(3x^2+1)$	$y' = 15\tan^2(3x^2+1).\left[1+\tan^2(3x^2+1)\right].6x$
19 b)	$y = \sqrt{x+\sqrt{x}}$	$y' = \dfrac{1}{2\sqrt{x+\sqrt{x}}}.\left(1+\dfrac{1}{2\sqrt{x}}\right)$
20 a)	$y = \sqrt{\tan x^2}$	$y = \dfrac{1}{2\sqrt{\tan x^2}}.(1+\tan^2 x^2).2x =$ $= \dfrac{x(1+\tan^2 x^2)}{\sqrt{\tan x^2}}$
20 b)	$y = \sqrt[3]{\dfrac{x-2}{x+2}}$	$y' = \dfrac{1}{3\sqrt[3]{\left(\dfrac{x-2}{x+2}\right)^2}}.\dfrac{(x+2)-(x-2)}{(x+2)^2} =$ $= \dfrac{4}{3(x+2)^2}\sqrt[3]{\left(\dfrac{x+2}{x-2}\right)^2}$