

Resuelve, tomando logaritmos, estas ecuaciones:

a)  $\frac{1}{e^x} = 27$

b)  $e^{x-9} = \sqrt{73}$

c)  $2^x \cdot 3^x = 81$

d)  $\frac{2^x}{3^{x+1}} = 1$

a)  $\frac{1}{e^x} = 27 \rightarrow \frac{1}{27} = e^x \rightarrow \ln \frac{1}{27} = \ln e^x$

$$x = \ln \frac{1}{27} = \ln 1 - \ln 27 = 0 - \ln 27 \rightarrow x \approx -3,296$$

b)  $e^{x-9} = \sqrt{73} \rightarrow \ln e^{x-9} = \ln \sqrt{73}$

$$x - 9 = \frac{1}{2} \ln 73 \rightarrow x = 9 + \frac{\ln 73}{2} \rightarrow x \approx 11,145$$

c)  $6^x = 81; x \log 6 = \log 81$

$$x = \frac{\log 81}{\log 6} \approx 2,453$$

d)  $\frac{2^x}{3^x \cdot 3} = 1; \left(\frac{2}{3}\right)^x = 3; x \log \frac{2}{3} = \log 3$

$$x = \frac{\log 3}{\log 2 - \log 3} \approx -2,710$$

Resuelve las siguientes ecuaciones mediante un cambio de variable:

a)  $2^x + 2^{1-x} = 3$

b)  $2^{x+1} + 2^{x-1} = \frac{5}{2}$

c)  $8^{1+x} + 2^{3x-1} = \frac{17}{16}$

d)  $2^{2x} - 5 \cdot 2^x + 4 = 0$

e)  $9^x - 3^x - 6 = 0$

f)  $7^{1+2x} - 50 \cdot 7^x + 7 = 0$

a)  $2^x + \frac{2}{2^x} = 3$

$$z = 2^x \rightarrow z + \frac{2}{z} = 3; \quad z^2 + 2 = 3z$$

$$z^2 - 3z + 2 = 0; \quad z = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} = \begin{matrix} 2 \\ 1 \end{matrix}$$

$$2^x = 2 \rightarrow x_1 = 1; \quad 2^x = 1 \rightarrow x_2 = 0$$

b)  $2 \cdot 2^x + \frac{2^x}{2} = \frac{5}{2}; \quad 4 \cdot 2^x + 2^x = 5; \quad 2^x = 1$

$$x = 0$$

c)  $2^3 + 3x + 2^{3x-1} = \frac{17}{16}$

$$8 \cdot (2^x)^3 + \frac{(2^x)^3}{2} = \frac{17}{16} \rightarrow 2^x = z \rightarrow 128z^3 + 8z^3 = 17$$

$$(128 + 8)(z)^3 = 17; \quad (z)^3 = \frac{17}{136} = \frac{1}{8} \rightarrow z = \sqrt[3]{\frac{1}{8}} = \frac{1}{2} \rightarrow 2^x = \frac{1}{2}$$

$$x = -1$$

d)  $(2^x)^2 - 5 \cdot 2^x + 4 = 0$

$$2^x = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} = \begin{matrix} 4 \\ 1 \end{matrix}$$

$$x_1 = 0; \quad x_2 = 2$$

e)  $(3^x)^2 - 3^x - 6 = 0; \quad 3^x = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2} = \begin{matrix} 3 \\ -2 \end{matrix}$  (no vale)

$$x = 1$$

f)  $7 \cdot (7^x)^2 - 50 \cdot 7^x + 7 = 0; \quad 7^x = \frac{50 \pm 48}{14} = \begin{matrix} 7 \\ 1/7 \end{matrix}$

$$x_1 = -1; \quad x_2 = 1$$

Resuelve las ecuaciones:

a)  $\log(x^2 + 1) - \log(x^2 - 1) = \log \frac{13}{12}$

b)  $\ln(x - 3) + \ln(x + 1) = \ln 3 + \ln(x - 1)$

c)  $2\ln(x - 3) = \ln x - \ln 4$

d)  $\log(x + 3) - \log(x - 6) = 1$

a)  $\log \frac{x^2 + 1}{x^2 - 1} = \log \frac{13}{12}$

$$12x^2 + 12 = 13x^2 - 13; \quad 25 = x^2$$

$$x_1 = -5; \quad x_2 = 5$$

b)  $\ln(x^2 - 2x - 3) = \ln(3x - 3)$

$$x^2 - 2x - 3 = 3x - 3; \quad x^2 - 5x = 0$$

$$x = 5 \quad (x = 0 \text{ no vale})$$

c)  $\ln(x - 3)^2 = \ln \frac{x}{4}$

$$x^2 + 9 - 6x = \frac{x}{4}$$

$$4x^2 + 36 - 24x = x; \quad 4x^2 - 25x + 36 = 0$$

$$x = \frac{25 \pm 7}{8} = \begin{cases} 4 \\ 9/4 \end{cases} \quad (\text{no vale})$$

$$x = 4$$

d)  $\log \frac{x + 3}{x - 6} = 1$

$$x + 3 = 10x - 60; \quad 63 = 9x$$

$$x = 7$$

**Resuelve las ecuaciones:**

a)  $\log(x + 9) = 2 + \log x$

b)  $\log \sqrt{3x + 5} + \log \sqrt{x} = 1$

c)  $2(\log x)^2 + 7\log x - 9 = 0$

d)  $\log(x^2 - 7x + 110) = 2$

• Haz  $\log x = y$ .

e)  $\log(x^2 + 3x + 36) = 1 + \log(x + 3)$

f)  $\ln x + \ln 2x + \ln 4x = 3$

a)  $\log \frac{x+9}{x} = 2$

$$x + 9 = 100x; \quad 9 = 99x; \quad x = \frac{9}{99} = \frac{1}{11}$$

$$x = \frac{1}{11}$$

b)  $\frac{\log(x(3x+5))}{2} = 1; \quad 3x^2 + 5x - 100 = 0$

$$x = \frac{-5 \pm 35}{6} = \begin{cases} 5 \\ -40/6 \text{ (no vale)} \end{cases}$$

$$x = 5$$

c)  $\log x = \frac{-7 \pm \sqrt{49 + 72}}{4} = \frac{-7 \pm 11}{4} = \begin{cases} 1; & x_1 = 10 \\ -18/4 = -9/2; & x_2 = 10^{-9/2} \end{cases}$

d)  $x^2 - 7x + 110 = 100; \quad x^2 - 7x + 10 = 0$

$$x = \frac{7 \pm \sqrt{49 - 40}}{2} = \frac{7 \pm 3}{2} = \begin{cases} 5 \\ 2 \end{cases}$$

$$x_1 = 2; \quad x_2 = 5$$

e)  $\log \frac{x^2 + 3x + 36}{x + 3} = 1$

$$x^2 + 3x + 36 = 10x + 30; \quad x^2 - 7x + 6 = 0$$

$$x = \frac{7 \pm \sqrt{49 - 24}}{2} = \frac{7 \pm 5}{2} = \begin{cases} 6 \\ 1 \end{cases}$$

$$x_1 = 1; \quad x_2 = 6$$

f)  $\ln x + \ln 2x + \ln 4x = 3$

$$\ln(x \cdot 2x \cdot 4x) = 3$$

$$\ln(8x^3) = 3 \rightarrow 8x^3 = e^3 \rightarrow x^3 = \frac{e^3}{8}$$

$$x = \sqrt[3]{\frac{e^3}{8}} = \frac{e}{2} \rightarrow x = \frac{e}{2}$$