Ejerc REPASO | TEMA I

① a) A^{10}?

$$A = \begin{pmatrix} 0 & k & t \\ 0 & 0 & k \\ 0 & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & k & t \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & 0 & k^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad A^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

En gral $A^n = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ para $n \geq 3$.

$$A^{10} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

b) B^{-1}. Por la definición.

$$\begin{pmatrix} 1 & k & t \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} a+kd+tg & b+ek+th & c+kf+ti \\ d+kg & e+kh & f+ki \\ g & h & i \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$g = 0 \qquad d+kg = 0 \to d = 0 \qquad a+kd+tg = 1 \to a = 1$

$h = 0 \qquad e+kh = 1 \to e = 1 \qquad b+ek+th = 0 \to b = -k$

$i = 1 \qquad f+ki = 0 \to f = -k \qquad c+kf+ti = 0 \Rightarrow c = k^2 - t$

$$B^{-1} = \begin{pmatrix} 1 & -k & k^2 - t \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}$$

c) si $k = 0$ $\quad B^{10}$?

$$B = \begin{pmatrix} 1 & 0 & t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad B^2 = \begin{pmatrix} 1 & 0 & 2t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad B^3 = \begin{pmatrix} 1 & 0 & 3t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow B^{10} = \begin{pmatrix} 1 & 0 & 10t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(2) $A\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ A^n

$A^2 = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$ $A^3 = \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix}$ ... En qual $A^n = \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}$

$A^{22} - 12A^2 + 2A = \begin{pmatrix} 1 & 22a \\ 0 & 1 \end{pmatrix} - 12\begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix} + 2\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -9 & 0 \\ 0 & -9 \end{pmatrix} = -9I$

(3) $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 2 \end{pmatrix}$

$XC + A = C + A^2$

$XC = C + A^2 - A$

$X = (C + \underbrace{A^2 - A}_{0}) \cdot C^{-1}$

$A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = A$

$X = C \cdot C^{-1} = I$

$|C| \neq 0$ $\exists C^{-1}$ por ser $rg(C) = 3$.

$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(4) $\left.\begin{array}{l} 2A + 3B = M \\ -A + B = N \end{array}\right\}$ $\begin{array}{l} \cancel{2}A + 3B = M \\ -\cancel{2}A + 2B = 2N \\ \hline \end{array}$

$5B = M + 2N$

$B = \frac{M + 2N}{5} = \frac{\begin{pmatrix} 8 & 4 & 7 \\ 18 & 11 & -6 \\ 8 & 3 & 13 \end{pmatrix} + 2\begin{pmatrix} 9 & -2 & 6 \\ 17 & 1 & -10 \\ 9 & 4 & 13 \end{pmatrix}}{5} =$

$B = \begin{pmatrix} 26/5 & 0 & 19/5 \\ 25/5 & 13/5 & -26/5 \\ 26/5 & 11/5 & 39/5 \end{pmatrix}$

$\left.\begin{array}{l} 2A + 3\cancel{B} = M \\ 3A - \cancel{3B} = -3N \end{array}\right\}$

$5A = M - 3N$.

$A = \begin{pmatrix} -19/5 & 10/5 & -11/5 \\ -33/5 & 8/5 & 24/5 \\ -19/5 & -9/5 & -26/5 \end{pmatrix}$

5 a) $A=\begin{pmatrix}-1 & -2 & -2\\ 1 & 2 & 1\\ 0 & -1 & -1\end{pmatrix}$

$A^3-I=0$.

$A^2=\begin{pmatrix}-1 & 0 & 2\\ 1 & 1 & -1\\ -1 & -1 & 0\end{pmatrix}$ $\quad A^3=A^2.A=\begin{pmatrix}-1 & 0 & 2\\ 1 & 1 & -1\\ -1 & -1 & 0\end{pmatrix}\begin{pmatrix}-1 & -2 & -2\\ 1 & 2 & 1\\ 0 & -1 & -1\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$

Se cumple que $A^3-I=0$ $\quad\boxed{A^3=I}$

b) A^{13}

Si $A^3=I$ $\quad A^{12}=I$ $\quad A^{13}=A^{12}.A=I.A=\underline{A}$

c) $A^2X+I=A \rightarrow A^2X=A-I$ $\quad$ Como $A^3=I$. multiplico por A por la Izda.

$A^3X=A(A-I)$

$X=A(A-I)=A^2-A$.

$X=\begin{pmatrix}0 & 2 & 4\\ 0 & -1 & -2\\ -1 & 0 & 1\end{pmatrix}$

6 $A=\begin{pmatrix}0 & 1 & 1\\ 1 & 1 & 0\\ 1 & 0 & 0\end{pmatrix}$ $\quad B=\begin{pmatrix}6 & -3 & -4\\ -3 & 2 & 1\\ -4 & 1 & 5\end{pmatrix}$ $\quad I=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$

λ? si $(A-\lambda I)^2=B$.

$A-\lambda I=\begin{pmatrix}0 & 1 & 1\\ 1 & 1 & 0\\ 1 & 0 & 0\end{pmatrix}-\begin{pmatrix}\lambda & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{pmatrix}=\begin{pmatrix}-\lambda & 1 & 1\\ 1 & 1-\lambda & 0\\ 1 & 0 & -\lambda\end{pmatrix}$

$(A-\lambda I)^2=B \rightarrow \begin{pmatrix}-\lambda & 1 & 1\\ 1 & 1-\lambda & 0\\ 1 & 0 & \lambda\end{pmatrix}\begin{pmatrix}-\lambda & 1 & 1\\ 1 & 1-\lambda & 0\\ 1 & 0 & -\lambda\end{pmatrix}=\begin{pmatrix}6 & -3 & -4\\ -3 & 2 & 1\\ -4 & 1 & 5\end{pmatrix}$

$$\begin{pmatrix} -\lambda^2+2 & 1-2\lambda & -2\lambda \\ 1-2\lambda & 2-2\lambda+\lambda^2 & 1 \\ -2\lambda & \lambda & \lambda^2+1 \end{pmatrix} = \begin{pmatrix} 6 & -3 & -4 \\ -3 & 2 & 1 \\ -4 & 1 & 5 \end{pmatrix}$$

$$\left.\begin{array}{ll} -\lambda^2+2=6 & 2-2\lambda+\lambda^2=2 \\ 1-2\lambda=-3 & \lambda^2+1=5 \\ -2\lambda=4 & \end{array}\right\} \boxed{\lambda=2}$$

(7) $A=\begin{pmatrix} m & 0 & 0 \\ 0 & 0 & m \\ 0 & -1 & m+1 \end{pmatrix}$ $F_2 \longrightarrow F_3$.

a) $rg(A)$

$A=\begin{pmatrix} m & 0 & 0 \\ 0 & -1 & m+1 \\ 0 & 0 & m \end{pmatrix}$

- Si $m=0$ $rg(A)=1$ F_1 y F_3 nulas.
- Si $m\neq 0$ $Rg(A)=3$.

b) $A=\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$ $rg(A)=3$. $\exists A^{-1}$

$XA+A=2I$; $XA=2I-A \to X=(2I-A)A^{-1}=2A^{-1}-I$

$A^{-1}=\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$ $X=2\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}-\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}=\begin{pmatrix} -3 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & -2 & -1 \end{pmatrix}$

$$\boxed{X=\begin{pmatrix} -3 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & -2 & -1 \end{pmatrix}}$$

EJERC. REPASO $\boxed{\text{TEMA 2}}$

(1)

$$P(x) = \begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 3 & 3 & x & 3 \\ 3 & 3 & 3 & x \end{vmatrix} \underset{\substack{F_3 - xF_1 \to F_3 \\ F_4 - 3F_1 \to F_4}}{\overset{F_2 \to F_1 + F_2}{=}} \begin{vmatrix} x & 1 & 1 & 1 \\ 1-x & x-1 & 0 & 0 \\ 3-x^2 & 3-x & 0 & 3-x \\ 3-3x & 0 & 0 & x-3 \end{vmatrix} = (-1)^{1+3} \begin{vmatrix} 1-x & x-1 & 0 \\ 3-x^2 & 3-x & 3-x \\ 3-3x & 0 & x-3 \end{vmatrix}$$

$\downarrow$
C_3 hac. ceros

$$\xrightarrow{F_2 + F_3 \to F_3} = \begin{vmatrix} 1-x & x-1 & 0 \\ \cancel{3-x^2} & \cancel{3-x} & \cancel{3-x} \\ 6-3x-x^2 & 3-x & 0 \end{vmatrix} = -(3-x) \begin{vmatrix} 1-x & x-1 \\ 6-3x-x^2 & 3-x \end{vmatrix} =$$

$$= (x-3)\left[(1-x)(3-x) - (x-1)(6-3x-x^2)\right] = (x-3)(1-x)\left[3-x+6-3x-x^2\right] =$$

$(x-3)(1-x)(-x^2-4x+9)$

Por lo que $x=3$ $x=1$ son raíces de $P(x)$

(2)

$$|A| = \begin{vmatrix} x & 2x+1 & 3x+2 \\ x & 2x+3 & 3x+4 \\ x & 2x+5 & 3x+6 \end{vmatrix} = x \begin{vmatrix} 1 & 2x+1 & 3x+2 \\ 1 & 2x+3 & 3x+4 \\ 1 & 2x+5 & 3x+6 \end{vmatrix} =$$

- Los eltos de C_1 están multiplicados por $\underline{x} \to x \cdot |A|$
- $F_2 - F_1 \to F_2$ $F_3 - F_1 \to F_3$ el det. no varía si a una fila le sumamos una c.l de las demás

$$= x \begin{vmatrix} 1 & 2x+1 & 3x+2 \\ 0 & 2 & 2 \\ 0 & 4 & 4 \end{vmatrix} \to F_2 \text{ y } F_3 \text{ son proporcionales}$$

$\underline{\underline{|A| = 0.}}$

③ $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 7.$

$$\begin{vmatrix} 3a & 3b & 3c \\ a+p & b+q & c+r \\ -x+a & -y+b & -z+c \end{vmatrix} = 3\begin{vmatrix} a & b & c \\ a+p & b+q & c+r \\ -x+a & -y+b & -z+c \end{vmatrix} \xrightarrow[F_3-F_1\to F_3]{F_2-F_1\to F_2}$$

$$3\begin{vmatrix} a & b & c \\ p & q & r \\ -x & -y & -z \end{vmatrix} = -3\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -3.7 = -21$$

④

$$A = \begin{pmatrix} -k & 4 & 5 & 6 \\ -k & 1 & 2 & 3 \\ -k & -k & 0 & -1 \\ -k & -k & -k & -1 \end{pmatrix}$$ Si $|A| = 0$ la matriz no tiene inversa.

$$\begin{vmatrix} -k & 4 & 5 & 6 \\ -k & 1 & 2 & 3 \\ -k & -k & 0 & -1 \\ -k & -k & k & -1 \end{vmatrix} = -k\begin{vmatrix} 1 & 4 & 5 & 6 \\ 1 & 1 & 2 & 3 \\ 1 & -k & 0 & -1 \\ 1 & -k & k & -1 \end{vmatrix} = -k\begin{vmatrix} 1 & 4 & 5 & 6 \\ 0 & -3 & -3 & -3 \\ 0 & -k-4 & -5 & -7 \\ 0 & -k-4 & -k-5 & -7 \end{vmatrix} =$$

$$-k\begin{vmatrix} -3 & -3 & -3 \\ -k-4 & -5 & -7 \\ -k-4 & -k-5 & -7 \end{vmatrix} = 3k\begin{vmatrix} 1 & 1 & 1 \\ k+4 & 5 & 7 \\ k+4 & k+5 & 7 \end{vmatrix} \underset{F_3-7F_1}{\overset{F_2-7F_1}{=}} 3k\begin{vmatrix} 1 & 1 & 1 \\ k-3 & -2 & 0 \\ k-3 & k-2 & 0 \end{vmatrix} =$$

$$= 3k\begin{vmatrix} k-3 & -2 \\ k-3 & k-2 \end{vmatrix} = 3k\left[(k-3)(k-2) + 2(k-3)\right] = 3k(k-3)k =$$

$$= 3k^2(k-3) \qquad |A| = 0 < \begin{matrix} k=0 \\ k=3 \end{matrix}$$

• Si $k=0$ ó $k=3$ $\quad \nexists A^{-1}$

b) • Si $\underline{k=0}$ $|A| = 0$ $rg(A) < 4$.

Buscamos un menor de orden 3 no nulo.

$$\begin{vmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 0 & 0 & -1 \end{vmatrix} = -\begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix} = -3 \neq 0 \qquad rg(A) = 3.$$

• Si $k = 3$.

$$\begin{vmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ -3 & 0 & -1 \end{vmatrix} \neq 0 \qquad rg(A) = 3$$

(5)

$$M = \begin{pmatrix} 2 & 1 & -a \\ 2a & 1 & -1 \\ 2 & a & 1 \end{pmatrix} \qquad |M| = 2a(1-a^2)$$

$|M| = 0$ $a = 0$, $a = \pm 1$

• Si $a \neq 0$ y $a \neq \pm 1$

$rg(M) = 3$. $|M| \neq 0$ $\exists A^{-1}$

• Si $a = 0 \to \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} \neq 0$ $rg(M) = 2$. $\nexists A^{-1}$

• Si $a = 1 \to \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} \neq 0$ $rg(M) = 2$. $\nexists A^{-1}$

• Si $a = -1$ $\begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix} \neq 0$ $rg(M) = 2$. $\nexists A^{-1}$

b) M^{-1}

$$M = \begin{pmatrix} 2 & 1 & -2 \\ 4 & 1 & -1 \\ 2 & 2 & 1 \end{pmatrix} \qquad |M| = -12 \quad \exists M^{-1} \qquad [Adj(M)]^t = \begin{pmatrix} 3 & -5 & -1 \\ -6 & 6 & -6 \\ 6 & -2 & -2 \end{pmatrix}$$

$$M^{-1} = \frac{1}{|A|} \cdot [Adj(M)]^t = \begin{pmatrix} -3/12 & 5/12 & \frac{1}{12} \\ 6/12 & -6/12 & 6/12 \\ -6/12 & 2/12 & 2/12 \end{pmatrix}$$

6) a) $A = \begin{pmatrix} 2 & 2 & 1 \\ -1 & -1 & -1 \\ 2 & 4 & 3 \end{pmatrix}$ $\quad T = \begin{pmatrix} 2 & 4 & 1 \\ -1 & -3 & -1 \\ 1 & 2 & 1 \end{pmatrix}$

$|T| \neq 0 \rightarrow \exists T^{-1}$ $\quad T^{-1} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ -1 & 0 & 2 \end{pmatrix}$

$|T| = -1$

b) $A = T^{-1} B \cdot T$

$TAT^{-1} = B$ $\quad$ Aplic. determinantes

$|TAT^{-1}| = |B|$

$|T||A||T^{-1}| = |B|$

$|T| \cdot |A| \cdot \frac{1}{|T|} = |B| \implies |B| = |A| = 2$

c) $B = TAT^{-1} = \begin{pmatrix} 2 & 4 & 1 \\ -1 & -3 & -1 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 & 1 \\ -1 & -1 & -1 \\ 2 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ -1 & 0 & 2 \end{pmatrix}$

$= \begin{pmatrix} 2 & 4 & 1 \\ -1 & -3 & -1 \\ 2 & 4 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ -1 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$