Ejercicios hoja derivadas

50. $y = \frac{a+\sqrt{x}}{a-\sqrt{x}}$ $\quad y' = \frac{\frac{a-\sqrt{x}}{2\sqrt{x}} - \left(-\frac{a+\sqrt{x}}{2\sqrt{x}}\right)}{(a-\sqrt{x})^2} =$

$$= \frac{\frac{1}{2\sqrt{x}}(a-\sqrt{x}+a+\sqrt{x})}{(a-\sqrt{x})^2} = \frac{1 \cdot 2a}{2\sqrt{x}(a-\sqrt{x})^2} = \frac{a\sqrt{x}}{x(a-\sqrt{x})^2}$$

51. $y = \sqrt{1+\sqrt{x}}$ $\quad y = (1+x^{1/2})^{1/2}$

$$y' = \frac{1}{2}(1+x^{1/2})^{1/2-1} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{1+\sqrt{x}}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$$

$$= \frac{\sqrt{x}}{4x\sqrt{1+\sqrt{x}}}$$

52. $y = \left(\frac{x^2+2}{4x+2}\right)^2$ $\quad y' = 2 \cdot \left(\frac{x^2+2}{4x+2}\right) \cdot \frac{2x(4x+2) - 4(x^2+2)}{(4x+2)^2} =$

$$= \frac{2(x^2+2)(4x^2+4x-8)}{(4x+2)^3}$$

53. $y = \frac{x^6}{(3x+2)^2}$ $\quad y' = \frac{6x^5(3x+2)^2 - 2(3x+2)\cdot 3 \cdot x^6}{(3x+2)^{2\cdot 2}} =$

$$\frac{6x^5(3x+2)^2 - 6x^6(3x+2)}{(3x+2)^4} = \frac{3x+2\left(6x^5(3x+2) - 6x^6\right)}{(3x+2)^4} =$$

$$= \frac{12x^6 + 12x^5}{(3x+2)^3}$$

54. $y = \ln(1+x^2)$ $\quad y' = \frac{2x}{1+x^2}$

55. $y = \ln\left(\frac{3-5x}{2x+7}\right) = \ln(3-5x) - \ln(2x+7)$

$y' = \frac{-5}{3-5x} - \frac{2}{2x+7} = \frac{-10x-35-6+10x}{(3-5x)(2x+7)} = \frac{-41}{(3-5x)(2x+7)}$

56. $y = \ln\left(\frac{x}{x^2+4}\right) \cdot y' = \frac{\frac{x^2+4-2x\cdot x}{(x^2+4)^2}}{\frac{x}{x^2+4}} = \frac{(-x^2+4)(x^2+4)}{(x^2+4)^2 \cdot x} =$

$= \frac{-x^2+4}{x(x^2+4)}$

57. $y = \ln\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right) = \ln(1+\sqrt{x}) - \ln(1-\sqrt{x}) =$

$y' = \frac{1}{(1+\sqrt{x})2\sqrt{x}} - \frac{-1}{(1-\sqrt{x})2\sqrt{x}} =$

$\frac{1-\sqrt{x}+1+\sqrt{x}}{(1-x)\cdot 2\sqrt{x}} = \frac{2}{(1-x)2\sqrt{x}} = \frac{1}{(1-x)\sqrt{x}}$

58. $y' = \frac{2x}{1-x^4}$

59. $y = e^{-x^2} \qquad y' = -2x \cdot e^{-x^2}$

60. $y = (x^2+1)\cdot e^{2x} \Rightarrow y' = 2x \cdot e^{2x} + 2(x^2+1)\cdot e^{2x} = (2x^2+2x+1)\,e^{2x}$

61. $y = e^{\sqrt{x}} \qquad y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

62. $y = \frac{e^x}{1+e^x} \qquad y' = \frac{(1+e^x) - e^x \cdot e^x}{(1+e^x)^2}$

(66) $y = \ln^2 x =$

$y' = 2 \ln x \cdot \frac{1}{x} = \frac{2 \ln x}{x}$

(67) $y = \ln^3 \left(\frac{3x^2-1}{2x} \right)$

$y' = 3 \ln^3 \left(\frac{3x^2-1}{2x} \right) \cdot \frac{6x \cdot 2x - (3x^2-1) \cdot 2}{4x^2} = 3 \cdot \ln^2 \left(\frac{3x^2-1}{2x} \right) \cdot \frac{(6x^2+2)2x}{4x^2(3x^2-1)}$

$\frac{(3x^2-1)}{2x}$

$= 3 \ln^2 \left(\frac{3x^2-1}{2x} \right) \cdot \frac{6x^2+2}{2x(3x^2-1)} = 3 \ln^2 \left(\frac{3x^2-1}{2x} \right) \cdot \frac{6x^2+2}{6x^3-2x}$

(68) $y = \operatorname{sen}^2 x \qquad y' = 2 \operatorname{sen} x \cdot \cos x = \operatorname{sen} 2x$

(69) $y = \cos x^2 \rightarrow y' = -\operatorname{sen} x^2 \cdot 2x = -2x \cdot \operatorname{sen} x^2$

(70) $y = (\cos x)^x \qquad \ln y = x \ln \cos x \; ; \; \frac{y'}{y} = \ln(\cos x) + \frac{x \cdot (-\operatorname{sen} x)}{\cos x} =$

$y' = \left| \ln(\cos x) - x \underbrace{\left(\frac{\operatorname{sen} x}{\cos x} \right)}_{\operatorname{tg} x} \right| \cdot (\cos x)^x = \left| \ln(\cos x) - x \operatorname{tg} x \right| \cdot (\cos x)^x$

(71) $y = \operatorname{tg}^2 x \qquad y' = 2 \operatorname{tg} x \cdot \frac{1}{\cos^2 x} = 2 \operatorname{tg} x \cdot \sec^2 x$

(72) $y = \operatorname{sen}(x^2+3x) \qquad y' = \cos(x^2+3x) \cdot (2x+3)$

(73) $y = 4 \operatorname{sen}^2 x + e^{\pi} \operatorname{sen} x \qquad y' = 4 \cdot \underbrace{2 \operatorname{sen} x \cdot \cos x}_{\operatorname{sen} 2x} + e^{\pi} \cos x =$

$y' = 8 \cos x \left(\operatorname{sen} x + e^{\pi} \right)$