Ejercicios Resueltos de Números complejos

Calcula en forma binómica y representa gráficamente la solución:

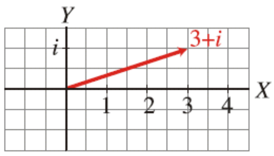
$$\frac{(4-2i)i^5}{1+i}$$

Solución:

$$\frac{(4-2i)i^5}{1+i}=\frac{(4-2i)i}{1+i}=\frac{4i-2i^2}{1+i}=\frac{4i+2}{1+i}=\frac{2+4i}{1+i}=\frac{(2+4i)(1-i)}{(1+i)(1-i)}=\frac{2-2i+4i-4i^2}{1-i^2}=\frac{2-2i+4i+4}{1+1}=$$

$$=\frac{6+2i}{2}=\frac{6}{2}+\frac{2i}{2}=3+i$$

Representación gráfica:



Escribe en forma polar los siguientes números complejos:

a) $1+\sqrt{3}i$ **b)** $\sqrt{3}+i$ **c)** $-1+i$

d) $5-12i$ **e)** $3i$ **f)** -5

a) $1+\sqrt{3}\,i = 2_{60°}$ b) $\sqrt{3}+i = 2_{30°}$ c) $-1+i = \sqrt{2}_{135°}$

d) $5-12i = 13_{292°\,37'}$ e) $3i = 3_{90°}$ f) $-5 = 5$

Escribe en forma binómica los siguientes números complejos:

a) $5_{(\pi/6)\text{ rad}}$ **b)** $2_{135°}$ **c)** $2_{495°}$

d) $3_{240°}$ **e)** $5_{180°}$ **f)** $4_{90°}$

a) $5_{(\pi/6)} = 5\left(\cos\frac{\pi}{6}+i\,\text{sen}\,\frac{\pi}{6}\right)=5\left(\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right)=\frac{5\sqrt{3}}{2}+\frac{5}{2}i$

b) $2_{135°} = 2(\cos 135° + i\,\text{sen}\,135°) = 2\left(-\frac{\sqrt{2}}{2}+i\,\frac{\sqrt{2}}{2}\right)=-\sqrt{2}+\sqrt{2}\,i$

c) $2_{495°} = 2_{135°} = -\sqrt{2}+\sqrt{2}\,i$

d) $3_{240°} = 3(\cos 240° + i\,\text{sen}\,240°) = 3\left(-\frac{1}{2}-i\,\frac{\sqrt{3}}{2}\right)=-\frac{3}{2}-\frac{3\sqrt{3}}{2}i$

e) $5_{180°} = -5$

f) $4_{90°} = 4i$

Halla las seis raíces sextas de 1. Represéntalas y exprésalas en forma binómica.

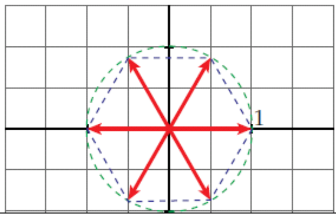
$\sqrt[6]{1} = \sqrt[6]{1_{0°}} = 1_{(360° \cdot k)/6} = 1_{60° \cdot k}; \quad k = 0, 1, 2, 3, 4, 5$

Las seis raíces son:

$1_{0°} = 1$ $\qquad 1_{60°} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ $\qquad 1_{120°} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$1_{180°} = -1$ $\qquad 1_{240°} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ $\qquad 1_{300°} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

Representación:



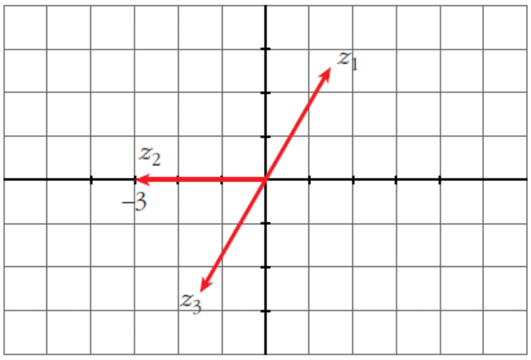
Resuelve la ecuación $z^3 + 27 = 0$. Representa sus soluciones.

$z^3 + 27 = 0 \rightarrow z = \sqrt[3]{-27} = \sqrt[3]{27_{180°}} = 3_{(180° + 360°\, n)/3} = 3_{60° + 120°\, n}; \quad n = 0, 1, 2$

$z_1 = 3_{60°} = 3(\cos 60° + i \operatorname{sen} 60°) = 3\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{3}{2} + \frac{3\sqrt{3}}{2}i$

$z_2 = 3_{180°} = -3$

$z_3 = 3_{240°} = 3(\cos 240° + i \operatorname{sen} 240°) = 3\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$



Calcula:

a) $\sqrt[3]{-i}$ **b)** $\sqrt[4]{-8 + 8\sqrt{3}\, i}$ **c)** $\sqrt{-25}$ **d)** $\sqrt[3]{\dfrac{-2 + 2i}{1 + \sqrt{3}i}}$

a) $\sqrt[3]{-i} = \sqrt[3]{1_{270°}} = 1_{(270° + 360°\, k)/3}$; $k = 0, 1, 2$

Las tres raíces son:

$1_{90°} = i \qquad 1_{210°} = -\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i \qquad 1_{330°} = \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i$

b) $\sqrt[4]{-8 + 8\sqrt{3}i} = \sqrt[4]{16_{120°}} = 2_{(120° + 360°\, k)/4} = 2_{30° + 90°\, k}$; $k = 0, 1, 2, 3$

Las cuatro raíces son:

$$2_{30°} = 2\left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) = \sqrt{3} + i$$

$$2_{120°} = 2\left(-\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right) = -1 + \sqrt{3}\, i$$

$$2_{210°} = 2\left(-\frac{1}{2} - i\,\frac{\sqrt{3}}{2}\right) = -1 - \sqrt{3}\, i$$

$$2_{300°} = 2\left(\frac{\sqrt{3}}{2} - i\,\frac{1}{2}\right) = \sqrt{3} - i$$

c) $\sqrt{-25} = \sqrt{25_{180°}} = 5_{(180° + 360°\, k)/2} = 5_{90° + 180°\, k}$; $k = 0, 1$

Las dos raíces son: $5_{90°} = 5i$; $5_{270°} = -5i$