$\boxed{6.50}$

$$r: \begin{cases} x+2y-z+3=0 \\ 3x+5y+2z-1=0 \end{cases} \qquad P(0,1,6)$$

$$r: \begin{cases} \vec{v}_r = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ 3 & 5 & 2 \end{vmatrix} = (9,-5,-1) \\ A_r(-1,0,2) \end{cases}$$

$$A_r \begin{cases} y=0 \\ \left.\begin{array}{l} x-z+3=0 \\ 3x+2z-1=0 \end{array}\right\} \longrightarrow \begin{array}{l} x=-1 \\ z=2 \end{array} \end{cases} \qquad P_r:(-1,0,2)$$

La recta es
$$r: \begin{cases} x=-1+9\lambda \\ y=-5\lambda \\ z=2-\lambda \end{cases}$$

$P \in r \rightarrow$ comprobamos $P \notin r$

$$\vec{A_rP} = (1,1,4)$$

$$d(P,r) = \frac{|\vec{A_rP} \times \vec{u}_r|}{|\vec{u}_r|}$$

$$\vec{A_rP} \times \vec{u}_r = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 4 \\ 9 & -5 & -1 \end{vmatrix} = (19,-14,37)$$

$$|\vec{A_rP} \times \vec{u}_r| = \sqrt{1926}$$

$$|\vec{u}_r| = \sqrt{107} \qquad d(P,r) = \frac{\sqrt{1926}}{\sqrt{107}} = \sqrt{18} = 3\sqrt{2}\ u$$

Si paso la recta r directamente a paramétrica obtengo el pto A_r y el vector.

$z=\lambda$

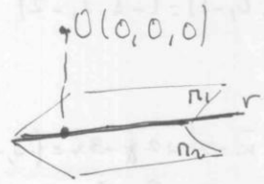
$$\left.\begin{array}{l} x+2y=\lambda-3 \\ 3x+5y=-2\lambda+1 \end{array}\right\}$$

$y=-10+5\lambda$

$x=17-9\lambda$

$$r: \begin{cases} x=17-9\lambda \\ y=-10+5\lambda \\ z=\lambda \end{cases}$$

6.57



$\pi_1: x+2y+z+4=0$

$\pi_2 \begin{cases} P(1,1,1) \\ Q(1,2,3) \\ R(2,0,0) \end{cases} \rightarrow \begin{matrix} \vec{PQ}=(0,1,2) \\ \vec{PR}=(1,-1,-1) \end{matrix}$

$$\pi_2: \begin{vmatrix} 0 & 1 & x-1 \\ 1 & -1 & y-1 \\ 2 & -1 & z-1 \end{vmatrix}$$

$$\boxed{\pi: x+2y-z-2=0}$$

$$r: \pi_1 \cap \pi_2 = \begin{cases} x+2y+z+4=0 \\ x+2y-z-2=0 \end{cases}$$

$$\vec{u_r} = (1,2,1)\times(1,2,-1) = (-4,2,0)$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 1 & 2 & -1 \end{vmatrix} = -4\vec{i}+2\vec{j} = (-4,2,0)$$

$$A_r \begin{cases} \boxed{y=0} & \left.\begin{matrix} x+z=-4 \\ x-z=2 \end{matrix}\right\} \\ & 2x=-2 \quad \boxed{x=-1} \\ & -1-z=2 \\ & -z=3 \Rightarrow z=-3 \end{cases}$$

$$A_r(-1,0,-3)$$

$$|\vec{OA}\times\vec{u_r}| = |(-1,0,-3)\times(-4,2,0)|$$

$$d(O,r) = \frac{|\vec{OA}\times\vec{u_r}|}{|\vec{u_r}|} =$$

$$\vec{OA}\times\vec{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & -3 \\ -4 & 2 & 0 \end{vmatrix} = -2\vec{k}+12\vec{j}+6\vec{i} = (6,12,-2)$$

$$\vec{u_r} = \sqrt{(-4)^2+2^2} = \sqrt{20}$$

$$|\vec{OA}\times\vec{u_r}| = \sqrt{6^2+12^2+(-2)^2} = 2\sqrt{46}$$

$$d(O,r) = \frac{2\sqrt{46}}{\sqrt{20}} = \frac{2\sqrt{46}}{2\sqrt{5}} = \sqrt{\frac{46}{5}}\ u$$

52 $P(1,0,3)$ $\quad r: \begin{cases} x+y=0 \\ 2x-z=0 \end{cases}$

$$d(P,r) = \frac{|\vec{AP} \times \vec{u_r}|}{|\vec{u_r}|}$$

$\vec{u_r} = (1,1,0)\times(2,0,-1) = (-1,1,-2)$

$A_r(0,0,0)$

$$d(P,r) = \frac{|(1,0,3)\times(-1,1,-2)|}{\sqrt{6}}$$

$$|\vec{AP}\times\vec{u_r}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 3 \\ -1 & 1 & -2 \end{vmatrix} = \vec{k} - 3\vec{j} + 2\vec{j} - 3\vec{i} = (-3,-1,1)$$

$$= \frac{\sqrt{11}}{\sqrt{6}} = \boxed{\frac{\sqrt{66}}{6}\, u}$$

otra forma

$\Pi \perp r$ que pasa por P

$\vec{u_r} = (-1,1,-2) = \vec{n_\Pi}$

$-x+y-2z+D=0 \xrightarrow{P(1,0,3)} -1+0-6=-D \quad \boxed{D=7}$

$\Pi: -x+y-2z+7=0 \simeq x-y+2z-7=0$

$r \cap \Pi \begin{cases} x+y=0 \\ 2x-z=0 \\ x-y+2z-7=0 \end{cases} \quad Q\left(\frac{7}{6}, -\frac{7}{6}, \frac{7}{3}\right) \quad P(1,0,3)$

$$d(P,r) = d(P,Q) = \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{7}{6}\right)^2 + \left(\frac{2}{3}\right)^2} = \frac{\sqrt{66}}{6}\, u$$

6.54 $Y \begin{cases} x=0 \\ z=0 \end{cases}$ $\quad \vec{u_s} = (0,1,0) \quad O(0,0,0)$

$s: \frac{x}{0} = \frac{y}{1} = \frac{z}{0}.$

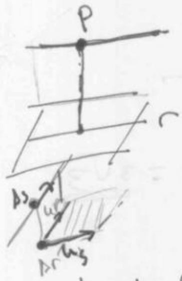
$r: \begin{cases} A(1,0,0) \\ B(0,1,1) \end{cases} \quad \vec{AB} = \vec{u_r} = (-1,1,1)$

Posición Relativa

$$\begin{array}{ccc} \vec{u_r} & \vec{u_s} & \vec{OA} \end{array}$$
$$\begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

M, M'

$rg\, M = 2$, $rg\, M' = 3$ } se cruzan



$$\boxed{d(r,s) = d(P_s, \Pi)}$$

$$\Pi \begin{cases} \vec{v}_\Pi = (-1,1,1) = \vec{u_r} \\ A(1,0,0) \\ \vec{u}_s = \vec{v}_\Pi = (0,1,0) \end{cases}$$

$$\begin{vmatrix} -1 & 0 & x-1 \\ 1 & 1 & y \\ 1 & 0 & z \end{vmatrix} = -z - x + 1 = 0 \qquad \underline{z + x = 1}$$

$\Pi: x + z - 1 = 0.$

$P(0,0,0)$

$$d(P,\Pi) = \frac{|-1|}{\sqrt{1^2+1^2+0}} = \frac{\sqrt{2}}{2}.$$

$$\boxed{d(P,\Pi) = \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2+B^2+C^2}} =}$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

Por la fórmula: $\vec{OA}_r(1,0,0)$

$$\frac{|\det(\vec{OA}, \vec{u}, \vec{v})|}{|\vec{u} \times \vec{v}|} = \frac{|1|}{\sqrt{1^2+1^2}} = \frac{\sqrt{2}}{2}$$

$$\vec{u_r} \times \vec{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\vec{k} - \vec{i} = (-1, 0, -1)$$

56

$$r: \begin{cases} x+3z-2=0 \\ y-4z=0 \end{cases} \qquad s: \begin{cases} x-2z-1=0 \\ y+z-3=0 \end{cases}$$

$\vec{u}_r = (1,0,3) \times (0,1,-4) = (-3,4,1)$

$\vec{u}_s = (1,0,-2) \times (0,1,1) = (2,-1,1)$

1º posic. relativa

$$\left(\begin{array}{cc|c} -3 & 2 & -1 \\ 4 & -1 & 3 \\ 1 & 1 & 0 \end{array}\right)$$

M

$|M'| \neq 0$

$$\left.\begin{array}{l} rg(M)=2 \\ rg(M')=3 \end{array}\right\} \text{Se cruzan.}$$

2º Distancia.

$$d(r,s) = \frac{|\det(\vec{u}_r, \vec{u}_s, \vec{P_rP_s})|}{|\vec{u}_r \times \vec{u}_s|}$$

$$\vec{u}_r \times \vec{u}_s = \begin{vmatrix} i & j & k \\ -3 & 4 & 1 \\ 2 & -1 & 1 \end{vmatrix} = (5,5,-5)$$

$$\begin{vmatrix} -3 & 4 & 1 \\ 2 & -1 & 1 \\ -1 & 3 & 0 \end{vmatrix} = \underline{10}$$

Paramétricas

$$r: \begin{cases} x = -3\lambda+2 = 2-3\lambda \\ y = 4\lambda = 4\lambda \\ z = \lambda = \lambda \end{cases}$$

$P_r = (2,0,0)$

$\vec{u}_r = (-3,4,1)$

$$s: \begin{cases} x = 2z+1 \\ y = -z+3 \\ z = \mu \end{cases} \qquad \begin{array}{l} x = 1+2\mu \\ y = 3-\mu \\ z = \mu \end{array}$$

$P_s\ (1,3,0)$

$\vec{u}_s = (2,-1,1)$

$\vec{P_rP_s} = (-1,3,0)$

$$d(r,s) = \frac{10}{\sqrt{5^2+5^2+5^2}} = \frac{10}{\sqrt{75}} = \frac{2\sqrt{3}}{3}\ \underline{\underline{u}}$$

$\boxed{60}$

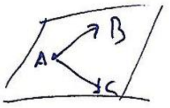
a) $A(-3,4,0)$, $B(3,6,3)$, $C(-1,2,1)$

$\vec{AB} = (6,2,3)$

$\vec{AC} = (2,-2,6)$

$\vec{n}_\Pi = \vec{AB} \times \vec{AC} = (8,0,-16)$

$\vec{n}_\Pi = (1,0,-2)$



$\Pi: Ax+By+Cz+D=0$

$\Pi: x-2z+D=0 \xrightarrow{P(-3,4,0)} -3+D=0 \quad \underline{\underline{D=3}}$

$\Pi: x-2z+3=0$

b) $\cos(\vec{AB},\vec{AC}) = \dfrac{(6,2,3)\cdot(2,-2,1)}{\sqrt{49}\ \sqrt{9}} = \dfrac{11}{21}$

$\cos(\vec{BA},\vec{BC}) = \dfrac{(-6,-2,-3)\cdot(-4,-4,-2)}{\sqrt{49}\ \sqrt{36}} = \dfrac{38}{42} = \dfrac{19}{21}$

$\cos(\vec{CA},\vec{CB}) = \dfrac{(-2,2,-1)\cdot(4,4,2)}{\sqrt{9}\cdot\sqrt{36}} = -\dfrac{2}{18} = -\dfrac{1}{9}$

c) $A_{Triáng} = \dfrac{1}{2}\left|\vec{AB}\times\vec{AC}\right| = \dfrac{1}{2}\sqrt{8^2+(-16)^2} = \underline{\underline{4\sqrt{5}\ u^2}}$

$\boxed{61}$

a) $\Pi: x+y+z=4$, $\Pi': x-z=0$, $\Pi'': x+y=3$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & 0 & -1 & 0 \\ 1 & 1 & 0 & 3 \end{array}\right) \xrightarrow{F_3-F_1} \begin{pmatrix} 1 & 1 & 1 & 4 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

$z=1$

$x-z=0 \qquad \underline{\underline{x=z=1}}$

$x+y+z=4 \rightarrow y=2$

$\boxed{P(1,2,1)}$

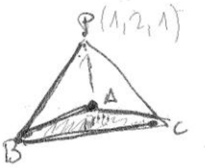
b) $\Pi\cap\Pi'' \begin{cases} x+y+z=4 \\ x=0 \end{cases} \qquad r: \begin{cases} x=0 \\ y=\lambda \\ z=4-\lambda \end{cases}$

$\Pi' : \Pi'' \begin{cases} x-z=0 \\ x=0 \end{cases} \qquad s: \begin{cases} x=0 \\ y=\mu \\ z=0 \end{cases}$

Π'' y Π''' $\left|\begin{array}{l} x+y=3 \\ x=0 \end{array}\right.$ $\rightarrow$ $t: \left\{\begin{array}{l} x=0 \\ y=3 \\ z=\beta \end{array}\right.$

c) $r \cap s \left\{\begin{array}{l} 0=0 \\ \lambda=\mu \\ 4-\lambda=0 \end{array}\right.$ $A(0,4,0)$ $\quad r \cap t: \left\{\begin{array}{l} 0=0 \\ \lambda=3 \\ 4-\lambda=\beta \end{array}\right.$ $B(0,3,1)$

$t \cap s \left\{\begin{array}{l} 0=0 \\ \mu=3 \\ \beta=0 \end{array}\right.$ $C(0,3,0)$



$A_{triang} = \frac{1}{2}\left(\vec{AB} \times \vec{AC}\right)$

$\left.\begin{array}{l} \vec{AB} = (0,-1,1) \\ \vec{AC} = (0,-1,0) \end{array}\right\}$. $\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1 & 1 \\ 0 & -1 & 0 \end{vmatrix} = \vec{i} = (1,0,0)$

$S_{tetraedro} = \frac{1}{2} \cdot 1 = \underline{\frac{1}{2}\ u^2}$

$P(1,2,1)$

$h_{tetraedro} = d(P, \Pi''') = \dfrac{1}{\sqrt{1}} = \underline{1}$ $\quad \boxed{\Pi''': x=0}$

$V_{tetraedro} = \frac{1}{3} A_b \cdot h = \frac{1}{3} \cdot \frac{1}{2} \cdot 1 = \boxed{\frac{1}{6}\ u^3}$

6.62

$\Pi \left\{\begin{array}{l} A(-1,0,3) \\ B(2,1,-1) \\ C(-3,2,0) \end{array}\right.$ $\quad \vec{AB} = (3,1,-4)$ $\quad \vec{AC} = (-2,2,-3)$ $\longrightarrow$ $\begin{vmatrix} 3 & -2 & x+1 \\ 1 & 2 & y \\ -4 & -3 & z-3 \end{vmatrix} = 0$

$6z - 18 - 3x - 3 + 8y + 8x + 8 + 2z - 6 + 9y = 0$

$\Pi: 5x + 17y + 8z - 19 = 0$

$\downarrow$

otra forma

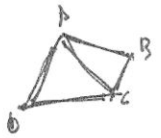
$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -4 \\ -2 & 2 & -3 \end{vmatrix} = (5,17,8)$

$\Pi: 5x + 17y + 8z + d = 0$ $\quad P(-1,0,3)$

$-5 + 0 + 24 + d = 0$ $\quad \underline{d=-19}$

$\Pi: 5x + 17y + 8z - 19 = 0.$

$$d(O,\pi)=\frac{|-19|}{\sqrt{5^2+13^2+8^2}}=\frac{19}{\sqrt{378}}\ u.$$

b) 

$$V=\frac{\det(\vec{OA},\vec{OB},\vec{OC})}{6}=\frac{1}{6}\begin{vmatrix}-1&0&3\\2&1&-1\\-3&2&0\end{vmatrix}=\frac{1}{6}\cdot 19=\boxed{\frac{19}{6}\ u^3}$$

65

$A(-2,-2,-3)\longrightarrow A'$ $\qquad \pi: 2x+y+z-3=0$

$r\perp\pi$ que pasa por A $\qquad r:\begin{cases}\vec{v}_r=\vec{n}_\pi=(2,1,1)\\ A(-2,-2,-3)\end{cases}$

$$r:\begin{cases}x=-2+2\lambda\\ y=-2+\lambda\\ z=-3+\lambda\end{cases}$$

$M\leftarrow r\cap\pi$ $\quad 2(-2+2\lambda)+(-2+\lambda)+(-3+\lambda)-3=0$

$6\lambda=12$ $\quad\boxed{\lambda=2}$ $\quad\boxed{M(2,0,-1)}$

pl $\quad \frac{x-2}{2}=2 \Rightarrow x=4-2=2$

$\frac{y-2}{2}=0 \Rightarrow y=2$ $\qquad \boxed{A'(6,2,1)}$

$\frac{z-3}{2}=-1 \Rightarrow z=1$

6.67 $r: \begin{cases} x-y=0 \\ 2x+z-1=0 \end{cases}$ $s: \begin{cases} x-z-2=0 \\ y-z-2=0. \end{cases}$ $\pi: \begin{cases} A(1,0,2) \\ B(2,1,1) \\ C(1,0,1 \end{cases}$

a) Ecuac. π.

$\pi: \begin{cases} \vec{AB}=(1,1,-1) \\ \vec{AC}=(0,0-1) \end{cases}$ $\vec{n}_\pi = \vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 0 & 0 & -1 \end{vmatrix} =$

$= -\vec{i} + \vec{j} = (-1,1,0)$

$\pi: -x+y+0z+d=0.$ $A(1,0,2) \Rightarrow d=1$

$-1 =$ $\pi: -x+y+1=0$ $\boxed{x-y-1=0}$ $\vec{n}_\pi = (-1,+1,0)$

b)

$r: \begin{cases} x=1 \\ y=\lambda \\ z=1-2x \end{cases}$ $r: \begin{cases} x=1 \\ y=\lambda \\ z=1-2\lambda \end{cases}$ $\vec{u}_r = (0,1,-2)$

$s: \begin{cases} x=z+2. \\ y=z+2 \end{cases}$ $\begin{cases} x=2+\lambda \\ y=2+\lambda \\ z=\lambda \end{cases}$ $\vec{u}_s = (1,1,1)$

r y π $\quad \vec{u}_r \cdot \vec{n}_\pi = 0 \quad r // \pi. \rightarrow$ condic. de paralelismo

$\vec{u}_r \cdot \vec{n}_\pi = (0,1,-2)(-1,1,0) \neq 0$ No son // $\boxed{\text{Se cortan}}$ r no está contenida en π.

s y π $\quad \vec{u}_s \cdot \vec{n}_\pi = (1,1,1)\cdot(-1,1,0) = -1+1=0 \nRightarrow$ $\boxed{\text{No se cortan}}$

$r\cap\pi.$ $\begin{cases} x=1 \\ y=\lambda \\ z=1-2\lambda \end{cases}$ $x-y-1=0$ $\quad 1-\lambda-1=0 \Rightarrow \boxed{\lambda=0.}$ $\boxed{P(1,0,1)}$ $\boxed{r\cap\pi}$

c) $\operatorname{sen}(\widehat{r,\pi}) = \cos(\vec{u}_r, \vec{n}_\pi) = \dfrac{|(0,-1,2)(-1,1,0)|}{\sqrt{1^2+2^2}\,\sqrt{1^2+1}} = \boxed{\dfrac{1}{\sqrt{10}}}$

d) $\boxed{\vec{u}_r \cdot \vec{n}_\pi = 0}$ $P_{(s)}$ ~~...~~ $\notin$ a $\pi \Rightarrow //$

$P_s(2,2,0)$.

$\pi': -x+y+d=0 \Rightarrow$

$\vec{n}_\pi(-1,1,0$ $-2+2+d=0$ $\boxed{d=0}$ $\boxed{\pi' = x-y=0}$

(6.69) $P(1,2,1)$ $P_2(2,3,1)$ $P_3(-1,4,3)$ $P_4(0,5,3)$

a) Calculo el $\vec{P_1P_2} = (1,1,0)$ $\vec{P_1P_3} = (-2,2,2)$

P_1, P_2, P_3

$$\begin{vmatrix} -2 & 1 & x-1 \\ 2 & 1 & y-2 \\ 2 & 0 & z-1 \end{vmatrix} = 0 \qquad \begin{aligned} &-2(z-1)+2(y-2)-2(x-1)-2(z-1)=0 \\ &-2z+2+2y-4-2x+2-2z+2=0 \\ &-2x+2y-4z+2=0 \\ &\boxed{\pi: x-y+2z-1=0.} \end{aligned}$$

$P_4 \in \pi$ $P_4(0,5,3)$

$-5+6-1=0$ (si) $\underline{P\in\pi}$ $\boxed{\text{Son coplanarios}}$

b) $\vec{P_1P_2}$ $\vec{P_1P_3}$ $\vec{P_2P_4}$ $\vec{P_3P_4}$ $\boxed{\vec{P_1P_2} \cdot \vec{P_1P_3} = 0}$

$A = |\vec{P_1P_2} \times \vec{P_1P_3}|$

73)

$\Pi: x+2y+3z=5$

a) $\Pi' \parallel \Pi \quad d(0,\Pi')=3.$ $\quad d(0,\Pi')=3=\dfrac{|D|}{\sqrt{1^2+2^2+3^2}}=3$

$\Pi': x+2y+3z+D=0$

$D=\pm 3\sqrt{14}$

$$\left.\begin{array}{l} x+2y+3z+3\sqrt{14}=0 \\ x+2y+3z-3\sqrt{14}=0 \end{array}\right\}$$

b) Distancia de P a Π.

al recta $\perp \Pi$ que pasa por O $\quad \vec{n}_\Pi=\vec{v}_r=(1,2,3)$

$O(0,0,0)$

$$r: \begin{cases} x=\lambda \\ y=2\lambda \\ z=3\lambda. \end{cases}$$

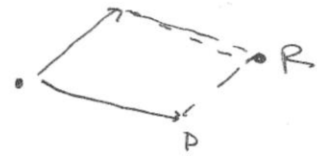
$r\cap\Pi \quad \lambda+2\cdot2\lambda+3\cdot3\lambda=5$

$14\lambda=5 \quad \lambda=\frac{5}{14}$

$$P\left(\frac{5}{14},\frac{10}{14},\frac{15}{14}\right)$$

5)

c)



$\vec{OQ}=(1,1,1)$

$\vec{OP}=\left(\frac{5}{14},\frac{10}{14},\frac{15}{14}\right)$

$$\text{Area}=|\vec{OQ}\times\vec{OP}|=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 1\\ \frac{5}{14} & \frac{10}{14} & \frac{15}{14}\end{vmatrix}=\left(-\frac{5}{14},\frac{10}{14},\frac{-5}{14}\right)$$

$$\text{Area}=\sqrt{\left(-\frac{5}{14}\right)^2+\left(\frac{10}{14}\right)^2+\left(-\frac{5}{14}\right)^2}=\frac{5}{14}\sqrt{6}\ u^2$$

$$\vec{OR}=\vec{OP}+\vec{OQ}=\left(\frac{19}{14},\frac{24}{14},\frac{29}{14}\right)$$