SOLUC DERIVADAS EJERC 15 – 22

Ejercicio 15.

a) Si $f(x) = \operatorname{sen}(3x+1)$.

$$f'(x) = 3\cos(3x+1)$$

b) Si $f(x) = \operatorname{sen}(x^3+1)$.

$$f'(x) = 3x^2\cos(x^3+1)$$

c) Si $f(x) = \operatorname{sen}^3(x^2+1)$. Luego

$$f'(x) = 3\operatorname{sen}^2(x^2+1)\cos(x^2+1)\,(2x)$$

d) Si $f(x) = \cos(\frac{x}{1-x})$.

$$f'(x) = -\operatorname{sen}(\frac{x}{1-x})\frac{1}{(1-x)^2}$$

e) Si $f(x) = \tan(1+2x^2+x^3)$,

$$f'(x) = \frac{1}{\cos^2(1+2x^2+x^3)}(4x+3x^2)$$

f) Si $f(x) = \sec(1-x^2) = \frac{1}{\cos(1-x^2)}$

$$f'(x) = \frac{-\operatorname{sen}(1-x^2)\,(-2x)}{\cos^2(1-x^2)}$$

Ejercicio 17.

a) Si $f(x) = \ln(x+\sqrt{x}+1)$.

$$f'(x) = \frac{1}{x+\sqrt{x}+1}(1+\frac{1}{2\sqrt{x}})$$

b) Si $f(x) = \ln(x^2+\operatorname{sen} x)$ $\quad f'(x) = \frac{1}{x^2+\operatorname{sen} x}(2x+\cos x)$

c) Si $f(x) = \ln(x^2\operatorname{sen} x)$.

$$f'(x) = \frac{1}{x^2\operatorname{sen} x}(2x\operatorname{sen} x + x^2\cos x)$$

d) Si $f(x) = \ln^2(1+\ln x)$.

$$f'(x) = 2\ln(1+e^x)\frac{1}{1+e^x}e^x$$

e) Si $f(x) = \ln^2(1+\ln x)$.

$$f'(x) = 2\ln(1+\ln x)\frac{1}{1+\ln x}\frac{1}{x}$$

f) Si $f(x) = \log_5(\frac{1}{1+\operatorname{sen} x})$.

$$f'(x) = \frac{1}{\ln 5}\frac{1}{\frac{1}{1+\operatorname{sen} x}}\frac{-\cos x}{(1+\operatorname{sen} x)^2} = -\frac{1}{\ln 5}\frac{\cos x}{1+\operatorname{sen} x}$$

Ejercicio 16.

a) Si $f(x) = e^{-5x+4x^2}$.

$$f'(x) = e^{-5x+4x^2}(-5+8x)$$

b) Si $f(x) = e^{x \operatorname{sen} x}$.

$$f'(x) = e^{x \operatorname{sen} x}(\operatorname{sen} x + x\cos x)$$

c) Si $f(x) = e^{1-\operatorname{sen}^2 x}$. Luego

$$f'(x) = e^{1-\operatorname{sen}^2 x}(-2\operatorname{sen} x\, \cos x)$$

d) Si $f(x) = 2^{\tan 3x}$.

$$f'(x) = 2^{\tan 3x}\ln 2\,\frac{3}{\cos^2 3x}$$

e) Si $f(x) = \left(\frac{3}{5}\right)^{x^2+3x}$,

$$f'(x) = \left(\frac{3}{5}\right)^{x^2+3x}(2x+3)\ \ln\frac{3}{5}$$

f) Si $f(x) = a^{\operatorname{sen} x+\cos x}$

$$f'(x) = a^{\operatorname{sen} x+\cos x}\ln a\,(\cos x - \operatorname{sen} x)$$

Ejercicio 18.

a) Si $f(x) = \operatorname{arc\,sen}(-x)$ $\quad f'(x) = -\frac{1}{\sqrt{1-(-x)^2}}$

b) Si $f(x) = \arctan(x^2)$ $\quad f'(x) = \frac{2\,x}{\sqrt{1+x^4}}$

c) Si $f(x) = \operatorname{arc\,sen}(\ln x + x)$.

$$f'(x) = \frac{1}{\sqrt{1-(\ln x+x)^2}}(\frac{1}{x}+1)$$

d) Si $f(x) = \operatorname{arc\,cos}(1-x)$.

$$f'(x) = \frac{-1}{\sqrt{1-(1-x)^2}}(-1)$$

e) Si $f(x) = \arctan(\operatorname{sen} x)$.

$$f'(x) = \frac{1}{1+(\operatorname{sen} x)^2}\cos x$$

f) Si $f(x) = \arctan(\ln x)$.

$$f'(x) = \frac{1}{1+(\ln x)^2}\frac{1}{x}$$

Ejercicio 19.

a) Si $f(x) = \ln^2(1+\cos x)^3$.

$$f'(x) = 2\ \ln(1+\cos x)^3\frac{1}{(1+\cos x)^3}3(1+\cos x)^2(-\operatorname{sen} x)$$

$$= -6\ \ln(1+\cos x)^3\frac{\operatorname{sen} x}{(1+\cos x)}$$

b) Si $f(x) = \operatorname{sen} x(1+\cos x)^3$.

$$f'(x) = \cos x(1+\cos x)^3 + \operatorname{sen} x\,3(1+\cos x)^2(-\operatorname{sen} x)$$

c) Si $f(x) = e^{1-\operatorname{sen} x}$.

$$f'(x) = -e^{1-\operatorname{sen} x}\cos x$$

d) Si $f(x) = 8^{x-\ln x}$.

$$f'(x) \ = \ 8^{x-\ln x}\cdot\ln 8\cdot(1-\frac{1}{x})$$

Ejercicio 21.

a) Si $f(x) = x^2\cdot\arctan x^{-1/2}$.

$$\begin{aligned} f'(x) &= 2x\cdot\arctan\frac{1}{\sqrt{x}} + x^2\cdot\frac{1}{1+(\frac{1}{\sqrt{x}})^2}\cdot\frac{-1}{2}x^{-3/2} \\ &= 2x\cdot\arctan\frac{1}{\sqrt{x}} - \frac{1}{2}\frac{\sqrt{x}}{1+(\frac{1}{\sqrt{x}})^2} \end{aligned}$$

b) Si $f(x) = x^x$. Aplicando logaritmos

$$\ln f(x) = x\ \ln x$$

$$\frac{f'(x)}{f(x)} = \ln x + x\cdot\frac{1}{x}$$

$$f'(x) = (\ln x + 1)\cdot x^x$$

Ejercicio 20.

a) Si $f(x)=\ln(1-\sqrt{x})^2$.

$$f'(x) = \frac{1}{(1-\sqrt{x})^2}2(1-\sqrt{x})\frac{-1}{2\sqrt{x}} = -\frac{1}{\sqrt{x}(1-\sqrt{x})}$$

b) Si $f(x)=\ln\sqrt{\frac{1+\tan x}{1-\tan x}}$.

$$\begin{aligned} f'(x) &= \frac{1}{\sqrt{\frac{1+\tan x}{1-\tan x}}}\cdot\frac{1}{2\sqrt{\frac{1+\tan x}{1-\tan x}}}\cdot \\ &\cdot\frac{\sec^2 x(1-\tan x)+(1+\tan x)\sec^2 x}{(1-\tan x)^2} \\ &= \frac{1-\tan x}{2(1+\tan x)}\cdot\frac{2\sec^2 x}{(1-\tan x)^2} \\ &= \frac{\sec^2 x}{(1+\tan x)(1-\tan x)} \end{aligned}$$

Ejercicio 22.

a) Si $f(x)=(\tan x)^{\operatorname{sen} x}$. Aplicando logaritmos

$$\ln f(x) = \operatorname{sen} x\,\ln\tan x$$

$$\frac{f'(x)}{f(x)} = \cos x\,\ln\tan x + \operatorname{sen} x\,\frac{\sec^2 x}{\tan x}$$

$$\frac{f'(x)}{f(x)} = \cos x\,\ln\tan x + \frac{1}{\cos x}$$

$$f'(x) = (\cos x\,\ln\tan x + \frac{1}{\cos x})\cdot(\tan x)^{\operatorname{sen} x}$$

b) Si $f(x)=e^x\cdot\sqrt[x]{x}$. Aplicando logaritmos

$$\ln f(x) = x + \frac{1}{x}\ln x$$

$$\frac{f'(x)}{f(x)} = 1 - \frac{1}{x^2}\ln x + \frac{1}{x}\frac{1}{x}$$

$$\frac{f'(x)}{f(x)} = 1 - \frac{\ln x}{x^2} + \frac{1}{x^2}$$

$$f'(x) = (1 - \frac{\ln x}{x^2} + \frac{1}{x^2})\cdot e^x\cdot\sqrt[x]{x}$$

Ejercicio 23. Siendo

$$f(x) = \begin{cases} x^3-1 & x\le 1 \\ a\,x+b & 1<x \end{cases}$$

- Para que sea continua en $x=1$

$$f(1^-)=0=f(1^+)=a+b \Longrightarrow a+b=0$$

- Para que sea derivable en $x=1$.

$$f'(x) = \begin{cases} 3\,x^2 & x<1 \\ a & 1<x \end{cases}$$

$$f'(1^-)=3=f'(1^+)=a \Longrightarrow \boxed{a=3}$$

Sustituyendo en la ecuación $a+b=0$, se tiene $\boxed{b=-3}$